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|  | **Midterms Notes** | 2nd June 2023 |

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**Each midterm is worth 30% of the course mark.**

**Syllabus:**

* ODE of the first order
  + Separation of variables
  + Integration factor
  + Variation of parameters
  + Exact equations
* ODE of higher order
  + ODE with Constant Coefficient

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**Separation of variables:**

Integrating both sides

**Integration factor:**

… (1)

Multiplying equation (1) by the Integration Factor we get,

By Product Rule of Differentiation, the LHS becomes

Integrating both sides

**Bernoulli’s equation:**

Dividing throughout by we get,

… (1)

Substituting in equation (1) we get,

Let and

… (2)

Multiplying equation (2) by the Integration Factor we get,

By Product Rule of Differentiation, the LHS becomes

Integrating both sides,

Resubstituting and and

**Variation of parameters:**

**Exact Equations:**

Condition for exactness 🡺

Solution 🡺

Where

It can be proved that since the ODE is exact then all terms of x will be cancelled out in the LHS

Resubstitute in

Solution is

**Non-exact equations:**

**ODE with constant coefficients**:

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First, let us consider the homogeneous case;

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Discussion:

Remark:

Any homogeneous ODE of order n has exactly n good/basis solutions such as y1, y2, …, yn and the general solution can be written as;

Now, let us consider n = 2;

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Using the remark, this ODE has two basis/good solutions (say y1 and y2)

Maybe one of them looks like

Let us check to see if we can find such a ‘λ’

Substitute in the ODE

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Now, there are 3 cases;

Case 1: ;

Case 2: ;

Case 3: ;

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| Separation of Variables | | |
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| 1 |  | In-Class |
| 2 |  | In-Class |
| 3 |  | In-Class |
| 4 |  | Homework |
| 5 |  | Homework |
| 6 |  | Homework |
| 7 |  | Homework |
| 8 |  | Advanced Engineering Mathematics 2nd Edition by Michael D. Greenberg |
| 9 |  |
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| Integration Factor | | |
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| Bernoulli’s equation | | |
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| Variation of Parameter | | |
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| Exact Equations | | |
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| Non-Exact Equations | | |
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| ODEs with constant coefficient | | |
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